

Large tensor-to-scalar ratio and low scale inflation

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It is plausible that the scalar density perturbations are created by a relatively low scale model of inflation which matches the observations of CMB anisotropy and excites Standard Model baryons and cold dark matter, but generates negligible gravity waves. Nevertheless a significantly large tensor perturbations can be observed if there exists a prior phase of high scale inflation separated by a matter or radiation dominated epoch. In this paper we provide a simple example where gravity waves generated at high scales can trickle through the horizon of the second phase of inflation and leave a distinct imprint in the spectrum of the tensor modes with a strong red tilt. A first phase of *assisted inflation* occurring at a high scale $H \sim 10^{13}$ GeV is followed by a second phase of *MSSM inflation* which happens at $H \sim 1$ GeV. The largest tensor-to-scalar ratio is then bounded by $r_{\text{observed}} \leq 0.8$ on the largest scales, roughly of the size of the horizon.

The positive detection of stochastic gravitational waves, especially at very large angular scales, is considered to be a strong support for the inflation paradigm. It is believed that the PLANCK satellite will be able to detect gravity waves if the tensor to scalar ratio is significant, i.e. $r \equiv \mathcal{T}/\mathcal{S} \sim 0.01 - 0.1$ [1]. It is however a daunting task to generate large tensor perturbations in a reasonable model of inflation which is embedded within particle physics [2] and in string inspired models of inflation [3]. In realistic models based on the minimal supersymmetric standard model (MSSM) [4, 5, 6], where the inflaton carries the standard model (SM) charges and eventually decays into the SM baryons and the cold dark matter, inflation occurs at very low scales and generated gravity waves are too small to be detectable in the future experiments.

A significant tensor to scalar ratio is obtained in chaotic inflation [8]. However in this class of models the vacuum expectation value (VEV) of the inflaton exceeds the Planck scale. Moreover, the inflaton is an absolute gauge singlet carrying no SM charges, thus rendering the model with no real connection to particle physics. A plausible scenario with sub-Planckian VEVs, called *assisted inflation*, arises when multi-scalar fields collectively drive inflation [9, 10]. Assisted inflation has found more realistic connection to particle physics [11] and string theory [13, 14]. It can generate a reasonable tensor-to-scalar ratio which could potentially be detected by the future

CMB experiments.

In spite of these successes, assisted inflation can not automatically explain the SM baryons and cold dark matter. The model can at best be embedded in a hidden sector, so far, whose couplings to the SM fields are essentially set by hand. This situation remains in string theory, as the inflation sector and the matter sector are often not the same [15].

Tensor perturbations in two field inflationary models have been studied in [7], where both the phases of inflation occur at super-Planckian VEVs. It was envisaged that a contribution to the scalar perturbations came from both phases of inflation, thus producing a break in the power spectrum for the temperature anisotropy. Similar break in power spectrum for tensor perturbations were studied in [22, 23].

In this paper our goal is to avoid inflation at super-Planckian VEVs while still generating significant tensor perturbations with a large tensor-to-scalar ratio. Secondly, we wish to generate the entire scalar perturbations during the second phase of inflation in order not to have any observable break in the power spectrum for the CMB temperature anisotropy. Third important point is that we wish to present a realistic model of inflation where reheating to the Standard Model baryons and cold dark matter is automatic and there are no uncertainties in the inflaton coupling to matter.

Here we aim to keep all of the virtues of an MSSM

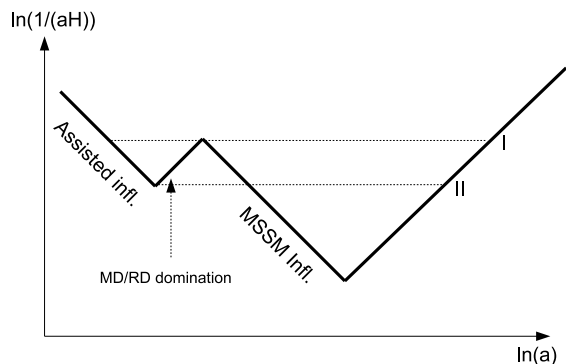


FIG. 1: Schematic diagram of the two phases of inflation separated by matter/radiation dominated epoch. Mode 1 skims through the horizon of the second phase of inflation. Note that assisted inflation happens at a high scale while MSSM inflation takes place at low scales.

based low scale inflation with direct connection to rich phenomenology [4, 5, 6], but modify the physics at higher energies, such that gravity waves can trickle past through the horizon of the MSSM inflation. The idea will be to have two phases of inflation where the first one contributes to the tensor modes, while the latter provides scalar density perturbations and successful reheating into the (dark) matter. We strictly assume that the entire CMB temperature anisotropy arises from the second phase of inflation, therefore, there is no break in its power spectrum. Moreover, the first phase of inflation can also set initial conditions for a low scale inflation as we will discuss briefly

In Fig. 1 we show a schematic diagram of the scenario, two inflationary phases separated by a matter/radiation dominated epoch. For the purpose of our discussion, without loss of generality, we will only concentrate on a matter dominated intermediate epoch. The largest observable scale is somewhere between the scale where the first phase of inflation, mode I, ends and the second starts, mode II. Here for simplicity we mainly concentrate on the case where the largest observable scale corresponds to the scale where the MSSM inflation starts, i.e. mode I in the figure.

There will be a clear prediction for the tensor modes in our case compared to the case where both of the tensor and scalar perturbations are generated by a single phase of inflation. The distinction is that we would see the tensor modes *only* on the largest angular scales, i.e. $l = 2$,

and there will be a sharp drop in the correlations involving the tensor component at smaller angular scales with a power dropping $\sim k^{-5}$. We will also see that scalar and tensor modes arising from the first phase of inflation are anti-correlated.

I. HIGHLIGHTS OF ASSISTED INFLATION AND GRAVITY WAVES

In order to illustrate our case, let us assume that a first phase of assisted inflation is driven by n scalar fields with an identical potential:

$$V = \sum_i m^2 \chi_i^2. \quad (1)$$

This is well studied in Ref. [16]. The scalar and tensor power spectrum, denoted by $\mathcal{P}_R^{\text{assist}}$ and $\mathcal{P}_g^{\text{assist}}$ respectively, are given by:

$$\mathcal{P}_R^{\text{assist}} \approx \frac{H^2 \sum_i \chi_i^2}{16\pi^2 M_P^4}, \quad (2)$$

$$\mathcal{P}_g^{\text{assist}} = \frac{2H^2}{\pi^2 M_P^2}, \quad (3)$$

$$r_{\text{assist}} = \frac{\mathcal{P}_g^{\text{assist}}}{\mathcal{P}_R^{\text{assist}}} \approx \frac{8}{\mathcal{N}}, \quad (4)$$

where r_{assist} is the tensor-to-scalar ratio and \mathcal{N} is the number of e-foldings between the time the relevant modes exit the horizon and the end of inflation.

The observed value of \mathcal{P}_R is set by COBE normalization, i.e. $\mathcal{P}_R \simeq 6 \times 10^{-11}$. Hence detectable gravity waves, i.e. $r \geq 0.1$, can be generated if ¹:

$$H_{\text{assist}} \simeq 10^{13} \text{ GeV}. \quad (5)$$

After inflation the χ fields will start oscillating coherently (we assume that they have the same amplitude of oscillations by virtue of identical mass scale). There will be a long wait until the next phase of inflation begins because MSSM inflation occurs at a low scale, i.e. $H_{\text{MSSM}} \sim 1 \text{ GeV}$ [4]. Since all χ fields are gauge singlets, it is conceivable that their couplings to matter are extremely weak, and hence they do not decay for $H > 1$

¹ For the purpose of illustration we will assume that $\mathcal{P}_R^{\text{assist}} \leq 6 \times 10^{-11}$.

GeV². Then the universe will evolve as in a matter dominated epoch for $H_{\text{MSSM}} < H < H_{\text{assist}}$.

During this intermediate epoch modes which exit the horizon in the last $\mathcal{N}_{\text{re-enter}}$ e-foldings of assisted inflation, i.e. between mode I and II in Fig. 1, will re-enter the horizon set by the MSSM inflation [27]:

$$\mathcal{N}_{\text{re-enter}} = \left(\frac{1-n}{2} \right) \ln \left(\frac{V_{\text{assist}}}{V_{\text{MSSM}}} \right) \simeq 10. \quad (6)$$

For $H_{\text{assist}} \sim 10^{13}$ GeV and $H_{\text{MSSM}} \sim 1$ GeV and $n = 2/3$ for matter domination the relevant mode which leaves before the end of assisted inflation horizon leads to $\mathcal{N}_{\text{re-enter}} \simeq 10$ e-foldings³. For example, the mode I in Fig. (1) leaves 10 e-foldings before the end of assisted inflation.

The crucial observation of this paper is that the tensor-to-scalar ratio in such a two phase inflationary model, where the tensor perturbations from the first phase skim through the horizon of the second phase of inflation (i.e. mode 1 in Fig. 1), is always bounded by

$$\begin{aligned} r_{\text{observed}} &\leq \frac{8}{\mathcal{N}_{\text{re-enter}}} \left(\frac{\mathcal{P}_{\mathcal{R}}^{\text{assist}}}{\mathcal{P}_{\mathcal{R}}^{\text{MSSM}}} \right), \\ &\leq 0.8 \left(\frac{\mathcal{P}_{\mathcal{R}}^{\text{assist}}}{\mathcal{P}_{\mathcal{R}}^{\text{MSSM}}} \right). \end{aligned} \quad (7)$$

This assumes that the entire scalar perturbations are generated by the second phase of inflation, but its contribution to tensor perturbations is negligible. This result is generic and does not depend on the details of either the first phase or the second phase of inflation. The numerical value only depends on the Hubble expansion rates for the two phases. In the most optimistic scenario when both the phases of inflations generate similar amplitude, spectral tilt and the running of the spectral tilt, the above bound can be saturated. On the other hand, if the amplitude of scalar perturbations from the second phase is dominant, then the bound is strict, $r_{\text{observed}} < 0.8$.

² The gauge singlet inflaton couples to the SM fermions and gauge fields via dimension-5 operators, i.e. non-renormalizable interactions [17], leading to a small decay rate. It can have a renormalizable coupling to the SM Higgs, but for an absolute gauge singlet the strength of such a coupling is not constrained by any symmetry argument. Moreover, such a coupling does not necessarily lead to a complete decay of the inflaton.

³ For radiation domination we have $n = 1/2$, which results in $\mathcal{N}_{\text{re-enter}} \simeq 15$.

Another important comment is that if there was a single phase of inflation driven by the assisted inflation alone, then the tensor-to-scalar ratio would be determined by Eq. (4). In this case the relevant scalar perturbations are produced at roughly $\mathcal{N} \sim 60$, yielding $r_{\text{assist}} \sim 0.13$, which is lower than the maximum value allowed in our scenario, see Eq. (7).

II. HIGHLIGHTS OF MSSM INFLATION AND SCALAR PERTURBATIONS

We now briefly discuss MSSM inflation (for details, see [4, 5, 6]). Inflation is driven by MSSM flat directions, which are classified by *gauge-invariant* combination of scalar fields in the theory. The potential along such a flat direction (after being minimized along the angular direction) is given by:

$$V = \frac{1}{2} m_\phi^2 \phi^2 - A \frac{\lambda_n \phi^n}{n M_{\text{P}}^{n-3}} + \lambda_n^2 \frac{\phi^{2(n-1)}}{M_{\text{P}}^{2(n-3)}}, \quad (8)$$

Here ϕ denotes the radial component of the flat direction field, and A is a positive definite quantity of dimension mass. If A and m_ϕ are related by $A^2 = 8(n-1)m_\phi^2$, there is a saddle point:

$$\phi_0 = \left(\frac{m_\phi M_{\text{P}}^{n-3}}{\lambda_n \sqrt{2n-2}} \right)^{1/(n-2)}. \quad (9)$$

where $V'(\phi_0) = V''(\phi_0) = 0$. The potential is very flat near ϕ_0 , and it is given by:

$$V_{\text{MSSM}} = \frac{(n-2)^2}{2n(n-1)} m_\phi^2 \phi_0^2. \quad (10)$$

As a result, if the flat direction field is in the vicinity of ϕ_0 (and has a sufficiently small kinetic energy), there will be an ensuing phase of inflation. The Hubble expansion rate during inflation is given by

$$H_{\text{MSSM}} = \frac{(n-2)}{\sqrt{6n(n-1)}} \frac{m_\phi \phi_0}{M_{\text{P}}}. \quad (11)$$

Inflation ends when $|\eta_{\text{inf}}| \sim 1$, where $\epsilon \equiv (M_{\text{P}}^2/2)(V'/V)^2$ and $\eta_{\text{inf}} \equiv M_{\text{P}}^2(V''/V)$ are the slow roll parameters. The number of e-foldings between the time when the observationally relevant perturbations are generated and the end of inflation follows [25, 27]:

$$\mathcal{N}_{\text{COBE}} \simeq 66.9 + \frac{1}{4} \ln \left(\frac{V_{\text{MSSM}}}{M_{\text{P}}^4} \right) \simeq 50. \quad (12)$$

Here we have used the fact that, due to efficient reheating [5], the energy density in the inflaton gets converted into radiation very quickly after the end of MSSM inflation. The amplitude of scalar perturbations thus produced is given by:

$$\delta_H \simeq \frac{1}{5\pi} \sqrt{\frac{2}{3}n(n-1)(n-2)} \left(\frac{m_\phi M_P}{\phi_0^2} \right) \mathcal{N}_{\text{COBE}}^2. \quad (13)$$

We remind that the scalar power spectrum $\mathcal{P}_{\mathcal{R}}$ is related to δ_H through $\mathcal{P}_{\mathcal{R}} = (4/25)\delta_H^2$.

MSSM inflation can produce up to 10^3 e-foldings of slow roll inflation [4, 5]. However the exact number of e-foldings in the slow roll regime depends on the initial VEV of the inflaton. Since in our case assisted inflation sets the initial conditions for the MSSM inflation, we are focusing on those Hubble patches where the inflaton VEV matches to provide $\mathcal{N}_{\text{COBE}}$. On the other hand, in those regions where this criteria is not met, it would be hard to see any trace of tensor modes on the largest angular scales.

For weak scale supersymmetry, i.e. $m_\phi \simeq 100$ GeV – 10 TeV, acceptable $\delta_H = 1.91 \times 10^{-5}$ and $0.92 \leq n_s < 1$ ⁴, compatible with the WMAP 3-year's data, are obtained if $n = 6$ and $\lambda \sim 1$ [4], or if $n = 3$ and $\lambda \sim 10^{-12}$ [6]⁵. Then from Eqs. (9,11), we have $\phi_0 \sim 10^{14}$ GeV and $H_{\text{MSSM}} \sim \mathcal{O}(1 \text{ GeV})$.

Another important point for an MSSM inflation is that the model parameters naturally accommodate thermal dark matter, i.e. successful inflation is compatible with the allowed regions of the parameter space which yield the correct abundance of neutralino dark matter [19].

Although we invoke assisted inflation for generating gravity waves, it also serves another useful purpose as it can set the initial conditions for the MSSM inflation [20]. The phase of assisted inflation will induce large quantum fluctuations to the flat direction since $m_\phi \ll H_{\text{assist}}$. The flat direction will make quantum jumps in this phase which accumulate in a random

walk fashion [8]: $d\langle\phi^2\rangle/dt = H^3/4\pi^2$. Then the required number of e-foldings of assisted inflation, which is required to push $\langle\phi^2\rangle$ to the vicinity of ϕ_0 follows $\mathcal{N}_{\text{assist}} = \sqrt{6\pi^2}\phi_0/m$ [20]. For $\phi_0 \sim 10^{14}$ GeV and $m \approx 10^{13}$ GeV, the required number of e-foldings turns out to be $\mathcal{N}_{\text{assist}} \sim 80$. This is quite feasible in assisted inflation with a sufficiently large number of fields [11, 12, 25]⁶.

III. ANALYSIS OF TENSOR AND SCALAR MODES

Now we wish to analyze the evolution of tensor and scalar modes from the first phase of inflation. Note that the phase of MSSM inflation requires at least 50 e-foldings in order to explain the observed scalar perturbations [4, 5]. We are interested in those tensor modes which just skim through the horizon of the MSSM inflation, i.e. modes that were of the horizon size when MSSM inflation began. They will be observable today if MSSM inflation lasted the minimum number of required e-foldings, i.e. $\mathcal{N}_{\text{COBE}} = \mathcal{N}_{\text{MSSM}} \simeq 50$.

These modes (tensor and scalar) left the horizon about $\mathcal{N}_{\text{re-enter}} \simeq 10$ e-foldings before the end of assisted inflation, see Eq. (6). Modes that left earlier will be well outside the horizon when MSSM inflation started, and hence today as well. On the other hand, modes that left later will re-enter the horizon before the onset of MSSM inflation and get suppressed. In this Section, we will discuss this issue in detail.

A. Brief summary of scalar perturbations

The evolution of the scalar curvature perturbations in the early universe can be followed by using the *gauge-invariant* Mukhanov variable u [21], which on flat hypersurfaces is related to the fluctuation of the scalar field, $\delta\phi$, simply by $u = a\delta\phi$. The curvature perturbation, \mathcal{R} ,

⁴ The spectral index is given by: $n_s = 1 - 4\sqrt{\Delta^2} \cot[\mathcal{N}_{\text{COBE}}\sqrt{\Delta^2}]$ [5, 18, 19]. For $0 \leq \Delta^2 \leq \frac{\pi^2}{4\mathcal{N}_{\text{COBE}}^2}$, the entire allowed range $0.92 \leq n_s \leq 1.0$ is covered.

⁵ The inflaton is a linear combinations of slepton or squark fields in the former case [4]. In the latter case, a linear combination of the Higgs, slepton and sneutrino fields play the role of the inflaton [6].

⁶ An extreme flatness of the MSSM inflaton potential ensures that the flat direction VEV remains virtually frozen during the intermediate stages.

is related to the variable u by ⁷

$$u = -z\mathcal{R}, \quad (14)$$

where the variable z is defined as $z \equiv a\dot{\phi}/H$. The Fourier modes of u , denoted by u_k , evolve according to [21]

$$\partial_\tau^2 u_k + \left(k^2 - \frac{\partial_\tau^2 z}{z}\right) u_k = 0, \quad (15)$$

where τ denotes conformal time. The power spectrum of the curvature perturbation is given by

$$\mathcal{P}_{\mathcal{R}}^{1/2}(k) = \sqrt{\frac{k^3}{2\pi^2}} \left| \frac{u_k}{z} \right|. \quad (16)$$

Deep inside the Hubble radius, when $k \gg (\partial_\tau^2 z)/z$, the solution of Eq. (15) tends to the free field Bunch-Davies vacuum solution (with arbitrary normalization)

$$u_k \propto \frac{1}{\sqrt{2k}} e^{-ik\tau}. \quad (17)$$

Well outside the Hubble radius, $k \ll (\partial_\tau^2 z)/z$, the growing solution is simply

$$u_k \propto z, \quad (18)$$

indicating that the curvature perturbation tends to a constant outside the Hubble radius. From these considerations the qualitative picture is clear: the vacuum modes are stretched during inflation until they exit the horizon, resulting in a constant curvature perturbation at super-Hubble scales.

B. Brief summary of tensor perturbations

For the tensor modes, the situation is similar to the scalar case but there are also important differences. The

⁷ Although in assisted inflation there are multiple fields, during the inflationary epoch there exists a unique late time attractor trajectory [9]. This ensures that the perturbations are determined solely by the Mukhanov variable corresponding to a single field,

equation determining the evolution of modes is now

$$v_k'' + \left(k^2 - \frac{a''}{a}\right) v_k = 0, \quad (19)$$

where v_k is related the gravity wave power spectrum by

$$P_g^{1/2} = \sqrt{\frac{k^3}{2\pi^2}} \left| \frac{v_k}{a} \right|. \quad (20)$$

Again, deep inside the horizon ($aH/k \ll 1$) we have the free field solution for a mode

$$v_k \propto \frac{1}{\sqrt{2k}} e^{-ik\tau} \quad (21)$$

which freezes after leaving the horizon ($aH/k \gg 1$), $v_k \propto a$.

C. Tensor and scalar modes for a generic fluid

In order to see the effect of the second phase of inflation on the modes that have been generated and left the horizon during the first phase, we rewrite the equations in terms of physical properties of the cosmic fluid, i.e. instead of considering the scalar field and its potential, ϕ and $V(\phi)$, we express Eq. (15) in terms of the equation of state of the fluid, ω . This allows us to study general fluids and their effect on the primordial power spectrum without the need to specify the potential.

Here we concentrate on fluids with $\omega > -1$, but a similar analysis is straightforward to carry out also when $\omega \leq -1$.

By defining the effective equation of state $p = \omega\rho$ and using the definitions of pressure and energy density for a scalar field with a canonical kinetic term, Eq. (15) can be written as

therefore, there is only adiabatic mode and no isocurvature (or entropy) mode, see [9, 16]. This is particularly true in our case since the masses of all the fields are the same.

$$u_k'' - \frac{1}{2}(1+3\omega)u_k' + \left[\left(\frac{k}{aH}\right)^2 - \frac{1}{2}(1-3\omega) - \frac{3}{4}\frac{1-\omega}{1+\omega}\omega' - \frac{1}{2}\frac{\omega''}{1+\omega} + \frac{1}{4}\left(\frac{\omega'}{1+\omega}\right)^2 \right] u_k = 0, \quad (22)$$

where $' \equiv d/d(\ln(a))$. This is a general result valid for

$\omega > -1$. In particular one can use Eq. (22) to study

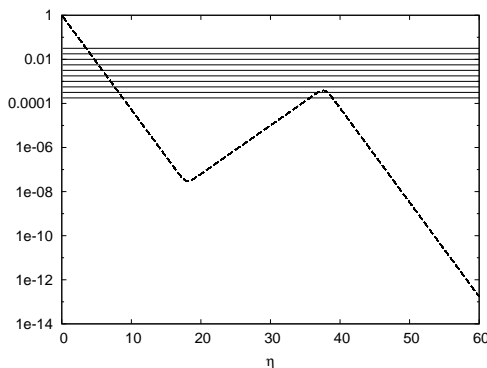


FIG. 2: Different k -modes and $(\ln(aH))^{-1}$ as a function of $\ln(a)$, $\Delta\eta = 3/2$.

the evolution of perturbations during transition periods in the early universe when the effective equation of state changes.

Similarly for gravity waves, we have

$$v_k'' - \frac{1}{2}(1+3\omega)v_k' + \left[\left(\frac{k}{aH} \right)^2 - \frac{1}{2}(1-3\omega) \right] v_k = 0. \quad (23)$$

Note the qualitative difference between the two equations, as long as the equation of state is constant, both u_k and u_k will evolve in a similar fashion. However, when the equation of state changes, i.e. when inflation stops (or a second phase of inflation starts), scalar and tensor modes will evolve differently.

IV. MODES SKIMMING THROUGH THE SECOND PHASE

In order to solve the mode equations numerically, one needs to specify the initial conditions. We adopt the approach also followed, for example, in [24, 25] and start the calculation for each mode deep inside the horizon, where it can be accurately described by the free field solution

$$u_k, v_k = \frac{1}{\sqrt{2k}} e^{-ik\tau}. \quad (24)$$

By integrating Eq. (22) and Eq. (23) for each k -mode, one can determine the produced spectrum via Eqs. (16,20). Due to the linearity of the equations, in practice it is useful to integrate the real and imaginary parts of u_k and v_k separately.

The equation of state changes from inflation, with $\omega = \omega_0 = -0.99$, to a matter dominated universe with $\omega =$

$\omega_1 = 0$ and then back to an inflationary epoch. The transition between the different phases is modeled with a hyperbolic tangent,

$$\omega(\eta) = \omega_0 + \frac{1}{2}(\omega_1 - \omega_0) \left(\tanh(\Delta\eta(\eta - \eta_0)) + \tanh(\Delta\eta(\eta_1 - \eta)) \right), \quad (25)$$

where η_0 marks the end of the first phase of inflation and the beginning of the matter/radiation dominated epoch, η_1 is when the second inflationary phase begins and $\Delta\eta$ is a parameter determining the smoothness of the transition.

In a realistic situation we expect that the transition from matter domination to MSSM inflation be smooth. The MSSM flat direction has frozen near the saddle point and carries a constant potential energy V_{MSSM} .

The oscillations of the χ fields which drive assisted inflation, see Eq. (1), behave as matter and dominate the universe during the intermediate stage. Their amplitude is redshifted by the Hubble expansion until the MSSM flat direction takes over. As a result, since there are no sudden changes in the potential of assisted or MSSM inflatons, the equation of state of the universe changes smoothly from that of matter domination to inflation.

The transition to the second phase of inflation can be accurately modeled as follows. Consider a universe filled with two fluids, $\rho_{1,2}$ with two different equations of state $\omega_{1,2}$ so that $\rho_{1,2} = \rho_{1,2}^0 \exp(-3(1+\omega_{1,2})\eta)$. The effective equation of state of the two fluid system is given by

$$\begin{aligned} w &= \frac{p_1 + p_2}{\rho_1 + \rho_2} = \frac{\omega_1 \rho_1 + \omega_2 \rho_2}{\rho_1 + \rho_2} \\ &= \frac{\omega_1 \rho_1^0 e^{-3(1+\omega_1)\eta} + \omega_2 \rho_2^0 e^{-3(1+\omega_2)\eta}}{\rho_1^0 e^{-3(1+\omega_1)\eta} + \rho_2^0 e^{-3(1+\omega_2)\eta}} \\ &= \frac{(\omega_2 - \omega_1)}{2} \tanh\left(\frac{3}{2}(\omega_2 - \omega_1)\eta - \frac{1}{2} \ln \frac{\rho_2^0}{\rho_1^0}\right) \\ &\quad + \frac{1}{2}(\omega_1 + \omega_2). \end{aligned} \quad (26)$$

Hence, for a matter dominated universe which later enters an inflationary phase dominated by an almost constant vacuum energy density, we see that $\Delta\eta = 3/2$.

In numerical calculations we will select $\eta_0 = 18$, $\eta_1 = 38$ and the numerical calculation is run until $\eta = 60$.

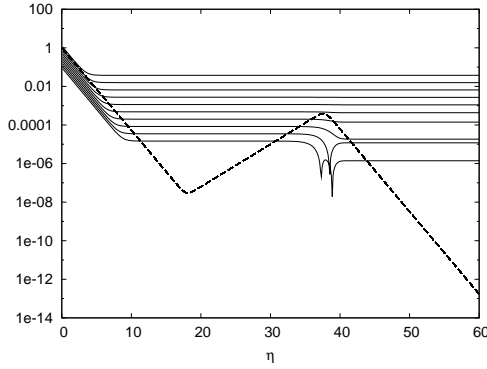


FIG. 3: The amplitude of the scalar modes (solid line), $|u_k/z|$, along with $(\ln(aH))^{-1}$ (dotted line) for $\Delta\eta = 3/2$ is plotted. Normalization is chosen arbitrary.

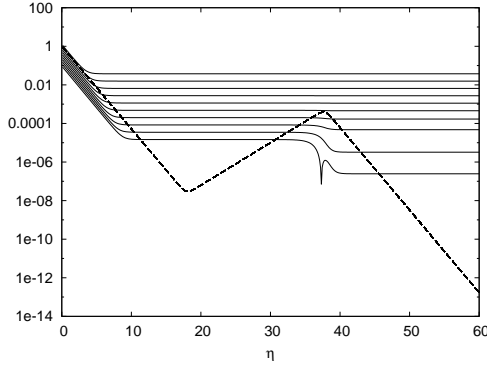


FIG. 4: The amplitude of the tensor modes (solid lines), $|v_k/a|$ along with $(\ln(aH))^{-1}$ (dotted line) is plotted, for a smooth transition from matter domination to (second phase of) inflation, $\Delta\eta = 3/2$. Normalization is chosen arbitrary.

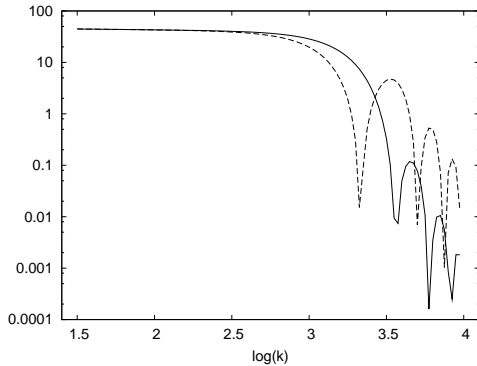


FIG. 5: The scalar (dotted line) and tensor (solid line) spectrum for $\Delta\eta = 3/2$ are plotted. Normalization is arbitrary and the mode that just grazes the horizon as the second phase of inflation starts has $\log_{10}(k) \approx 3.47$. One can clearly see that the scalar and tensor modes that re-enter the horizon before the second phase of inflation start decaying.

A. Evolution of scalar modes

In Fig. (2) we show $(\ln(aH))^{-1}$ along with a number of modes with different k as a function of η . A given mode exits/enters the horizon when $k = aH$, i.e. when the lines cross. Modes start deep inside the horizon, then freeze as they leave the horizon. As is clear from the figure, some of the modes then re-enter during the matter dominated epoch separating the inflationary periods. The modes shown here are sample of the modes used in calculating the resulting spectrum, i.e. we have used more modes.

In Fig. (3) we have shown the evolution of the amplitude of the scalar modes, $|u_k/z|$, as a function of η . We see that as the modes leave the horizon, they are frozen so that $u_k \propto z$, as expected. Modes that later re-enter the horizon get re-processed. Modes that are just entering the horizon as the second phase of inflation begins are re-processed and even modes that never re-enter the horizon undergo some evolution⁸.

B. Evolution of tensor modes and distinctive features

In Fig. (4) we have shown the evolution of the amplitude of the tensor modes, $|v_k/z|$, as a function of η . The tensor spectrum on the very large angular scales is solely given by the one arising from the assisted inflation.

The mode that just grazes the horizon, $k = k_c$, as the second inflationary phase begins, can be calculated by considering the evolution of $\ln(1/(aH))$. During the first inflationary phase H_{assist} is constant, and hence $\ln(1/(aH_{\text{assist}})) = -\eta - \ln(H_{\text{assist}})$. After the universe becomes matter/radiation dominated, $\ln(1/(aH)) = (1 + 3\omega)\eta/2 - \ln(H_{\text{assist}})$, where ω is the equation of state of the fluid dominating the universe between the inflationary phases. In the numerical calculation we have set, $H_{\text{assist}} = 1$, $\omega = 0$ so that when MSSM inflation starts at $\eta = 38$, $\ln(1/(aH)) = (38 - 18)/2 - 18 = -8$. Since a mode leaves the horizon when $k_c = aH$, $k_c = \exp(8)$ or $\log_{10}(k_c) \approx 3.47$.

⁸ During the transition it is conceivable to amplify the non-Gaussian parameter, f_{NL} , following Refs. [29]. We will come back to this issue in a separate publication.

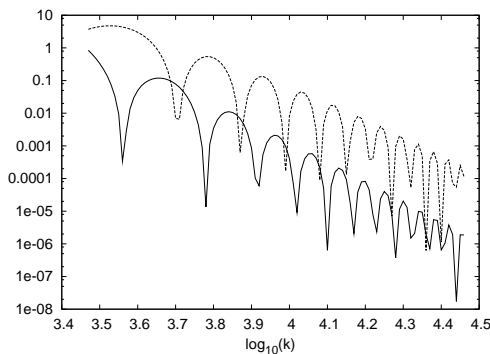


FIG. 6: Details of the tensor (solid lines) and scalar (dotted line) spectrum for $\Delta\eta = 3/2$. The largest mode corresponds to the critical mode grazing the horizon.

In Figs. (5, 6) we show the tensor and scalar power spectrum, $\mathcal{P}_{\mathcal{R}}(k)$ and $\mathcal{P}_g(k)$ respectively. From the figure one can read that both spectra are dropping with k with a large power. In the case of tensor perturbations, i.e. $\mathcal{P}_g(k) \sim (k/k_c)^{-6}$ in good agreement with analytical calculations [22]. The rate of decay is constant over the studied range of k , hence a general prediction of the scenario described here is that the tensor spectrum is very red with distinct bumps. The remnant scalar spectrum from the first phase of inflation also drops rapidly with increasing k . Furthermore, the tensor and scalar spectra exhibit anti-correlation so that an increase in scalar power means less power in the tensor power at the same scale (and vice versa).

If the whole observed scalar spectrum is due to the MSSM inflation and does not have features due to remnants from the assisted inflationary phase, then one expects that the tensor spectrum should be observable only at the very largest scales. For example, if the tensor-to-scalar ratio at $l(\log_{10}(k_c) = 3.47) = 2$ is of the order 0.8, then at $l = 10$, corresponding to $\log_{10}(k) = 4.47$ in Fig. 6), the tensor-to-scalar ratio is roughly $\sim 10^{-4}$.

So far we have assumed that the current horizon corresponds to the critical mode k_c . If we relax this assumption along with the details of the first inflationary phase, we have much more freedom in the scenario. For example, if the amplitude of scalar perturbations from the first phase of inflation is larger than the observed value, implying a higher scale for the assisted inflationary phase, one can have the current horizon correspond-

ing to a mode that has been sufficiently re-processed in between inflationary phases. Then one would still have the second phase of inflation producing scalar perturbations on all observable scales but now the horizon size can correspond to a mode which is smaller than the critical one, i.e. $\log_{10}(k) \approx 3.7$. For this mode one can read from Fig. (6) that the tensor-to-scalar ratio is amplified by an order of magnitude from its initial value. The other option is to consider scenarios where the observed scalar spectrum is a combination of the two inflationary phases. This naturally implies that there is a break in the scalar spectrum, therefore one would expect to see feature in the scalar spectrum. In all of the cases the main observable prediction remains, a very red tensor spectrum with distinct bumps observable at the largest scales. This can be associated with anti-correlated features in the scalar spectrum.

The anti-correlation between scalar and tensor spectra can be qualitatively understood as follows. First of all note that the frequencies of tensor and scalar modes are different, see Eqs. (22,23), they differ by the time variation in the equation of state as it changes from matter domination to inflation. This mainly affects modes whose wavelength is of the order of horizon size when the second phase of inflation began, i.e. modes to the left of Fig. (6). As a result the tensor and scale modes start oscillating at different times which leads to a phase difference between them. For modes which are deep inside the horizon at that time, i.e. modes to the right of Fig. (6), (k/aH) is large and the first term inside the brackets in Eqs. (22,23) dominate, leading to the same frequency for scalar and tensor modes. Hence the modes tend to become correlated to each other at larger values of k (larger values of multipoles, l). These are the features which distinguish our scenario with respect to the case where gravity waves arise *solely* from a single phase of inflation.

V. CONCLUSION

We have illustrated a simple example where we can generate observable gravity waves from assisted inflation at high scales $H_{\text{assist}} \approx 10^{13}$ GeV, while the origin of temperature anisotropy arises mainly from a low scale MSSM inflation $H_{\text{MSSM}} \sim 1$ GeV. The latter is also re-

sponsible for reheating the universe to the SM baryons and the cold dark matter.

The two phases of inflation are separated by a matter dominated epoch from coherent oscillations of the fields that drove assisted inflation. Being absolute gauge singlets, as usually assumed, these fields do not directly couple to matter, therefore, it is conceivable that they do not decay before the second phase of inflation kicks in.

The assisted inflation phase also creates the appropriate initial conditions for the MSSM inflation. This is a natural outcome due to the high scale of inflation, which imparts large quantum fluctuations to the MSSM flat direction that push it to the vicinity of the desired VEV, $\phi_0 \sim 10^{14}$ GeV, within ~ 80 e-foldings of assisted inflation.

The tensor-to-scalar ratio for a mode which is skimming through the second phase of inflation is bounded by Eq. (7). For $H_{\text{assist}} \approx 10^{13}$ GeV and $H_{\text{MSSM}} \sim 1$ GeV the

observed tensor-to-scalar ratio is given by $r_{\text{observed}} \leq 0.8$, observed only at the largest angular scales. The tensor perturbations have a distinctive feature shown in Figs. (5, 6), with a power spectrum that falls as $\sim k^{-6}$. This is in sharp contrast to the spectrum of gravity waves generated by a single phase of inflation alone, where such a re-processing of tensor modes would never occur. The future experiments may be able to distinguish between the two scenarios.

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